Exercise 14

If z is a complex number such that $|z| = 1$, that is, such that $z\bar{z} = 1$, compute

$$|1+z|^2 + |1-z|^2.$$

Solution

$$\begin{aligned}
|1+z|^2 + |1-z|^2 &= (1+z)(\overline{1+z}) + (1-z)(\overline{1-z}) \\
&= (1+z)(1+\bar{z}) + (1-z)(1-\bar{z}) \\
&= (1+\bar{z}+z+z\bar{z}) + (1-\bar{z}-z+z\bar{z}) \\
&= 2 + 2z\bar{z} \\
&= 2 + 2(1) \\
&= 4
\end{aligned}$$